Write your student exam number in the boxes								

Northern Beaches Secondary College

Manly Campus

2021 HIGHER SCHOOL CERTIFICATE

Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11 – 37, show relevant mathematical reasoning and/ or calculations

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Total marks: 100

Section I – 10 marks (pages 1-5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6-29)

- Attempt Questions 11 – 34
- Allow about 2 hour and 45 minutes for this section

MARKS	MC	Q11-16	Q17-21	Q22-25	Q26-28	Q29-31	Q32-34	TOTAL
STUDENT MARK								
MAXIMUM	10	16	15	13	15	14	17	100

Section I

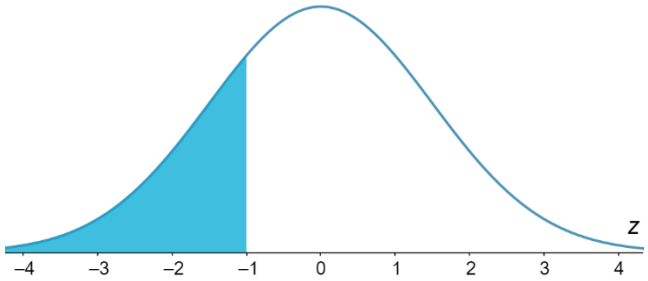
10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 –10

1. The graph shows the area under a normal distribution curve.



Which of the following is equivalent to the shaded area?

A. $P(z \geq -1)$

B. $P(z \geq 1)$

C. $1 - P(-1 \leq z \leq 1)$

D. $P(z \leq 1)$

2. Boxes are stacked in layers where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether. Which of the following is the correct expression for the number of boxes in the bottom layer?

A. $6n - 1$

B. $6n - 2$

C. $n + 5$

D. $n + 6$

3. Which of the following is the same as $\operatorname{cosec}(\pi + \theta)$?

A. $-\dfrac{1}{\sin\theta}$

B. $-\dfrac{1}{\cos\theta}$

C. $\dfrac{1}{\cos\theta}$

D. $\dfrac{1}{\sin\theta}$

4. What is the derivative of $y = 3^{2x+1}$?

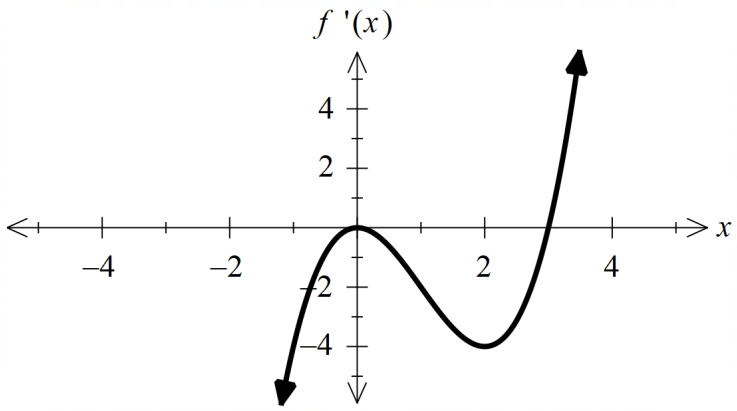
A. $y' = 2\ln 3 \cdot 3^{2x+1}$

B. $y' = \ln 3 \cdot 3^{2x+1}$

C. $y' = 2 \cdot 3^{2x+1}$

D. $y' = (2x+1) \cdot 3^{2x+1}$

5. The graph of $y = f\,'(x)$ is shown below. For what values of x is the function $f(x)$ decreasing?



A. $(0,2)$ B. $(3,\infty)$ C. $(-\infty,3)$ D. $(-\infty,0)\cup(0,3)$

6. The equation of the tangent to the curve $y = f(x)$ at the point (2,7) is $y = 5x - 3$. What is the equation of the tangent to the curve $y = f(x + 2) + 1$ at the point (0,8)?

A. $y = 5x - 12$

B. $y = 3x - 4$

C. $y = 5x + 8$

D. $y = 5x - 13$

7. Which of the following is not a probability density function?

A. $f(x) = \begin{cases} 3x^2 & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

B. $f(x) = \begin{cases} 2x^4 & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

C. $f(x) = \begin{cases} 4x^3 & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

D. $f(x) = \begin{cases} 2x & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$

8. Which expression is a term of the geometric series $3x-6x^2+12x^3-\ldots\ldots$

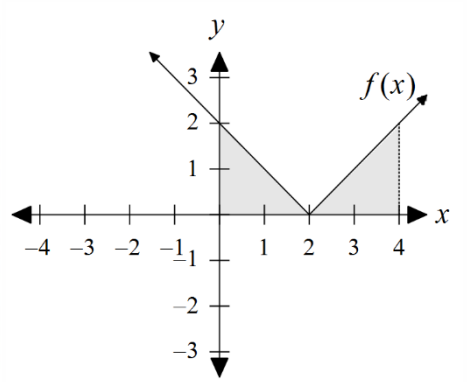
A. $3072x^{10}$

B. $-3072x^{10}$

C. $3072x^{11}$

D. $-3072x^{11}$

9. The graph of $f(x) = |x - 2|$ is shown.



Which of the following would give the area between $f(x)$ and the x-axis from $x = 0$ to $x = 4$, as shown by the shading on the graph?

A. $\left| \int_0^4 (x - 2)dx \right|$

B. $-\int_0^4 (x - 2)\, dx$

C. $2\int_0^2 (x - 2)\, dx$

D. $-2\int_0^2 (x - 2)\, dx$

10. Let $g(x) = 1 - f(x)$. If $\int_1^4 f(x)\, dx = 7$. What is the value of $\int_1^4 g(x)\, dx$?

A. -6 B. -4 C. 8 D. 10

End of Multiple Choice

Question 11 (2 marks)

A function $f(x)$ has the derivative $f'(x) = 4x^3 - 6x^2 + 5$ and $f(1) = 10$. **2**
Find the function.

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Question 12 (2 marks)

Evaluate the arithmetic series 3+8+13+18+…+308 **2**

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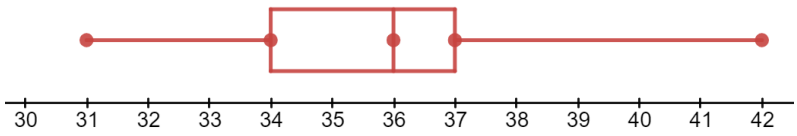
Question 13 (2 marks)

The box-plot represents the neck size, in centimetres, of 220 men.



a) How many men have a neck size of at least 36 cm? **1**

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Question 13 continues on page 8

b) Explain whether or not a neck size of 42 centimetres is an outlier. **1**

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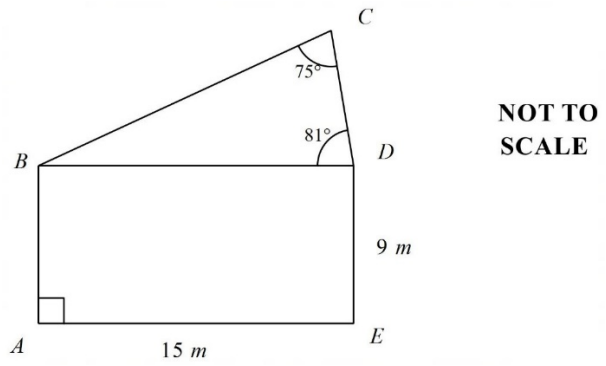
Question 14 (2 marks)

John owns a piece of land consisting of a rectangular section and a triangular section as shown in the diagram.



Calculate the length of CD to the nearest metre. **2**

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Question 15 (3 marks)

Prove that $\frac{\cos\theta}{1 + \sin\theta} + \tan\theta = \sec\theta$ **3**

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Question 16 (5 marks)

Differentiate with respect to x:

a) $\ln(x^2 - 5)$ **1**

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b) $\dfrac{\cos 3x}{x}$ **2**

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c) Show that $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ **2**

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Question 17 (4 marks)

The sum of the third term and the fifth term of an arithmetic sequence is 38.
The sum of the first twenty terms is 900.

Find the first term and the common difference of the sequence. **4**

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Question 18 (3 marks)

The graph of $y = \sqrt{x}$ undergoes the following sequence of transformations:

- Translation 3 units right
- Dilation horizontally by a factor of $\frac{1}{2}$
- Translation 1 unit down
- Dilation vertically by a factor of 4

Find the equation of the resulting graph. **3**

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Question 19 (4 marks)

Find the following integrals

a) $\int \frac{\sin(4x - 9)}{2}\, dx$ **2**

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b) $\int \frac{2e^x - 4x}{e^x - x^2}\, dx$ **2**

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Question 20 (2 marks)

Find the range of values of x for which the geometric series **2**

$$1 + \left(1 - \frac{x}{2}\right) + \left(1 - \frac{x}{2}\right)^2 + \left(1 - \frac{x}{2}\right)^3 \ldots\ldots$$

has a limiting sum.

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Question 21 (2 marks)

Results of a mathematics test are normally distributed with a mean of 60 and standard deviation of 14.

Approximately what percentage of students received a mark between 46 and 88? **2**

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Question 22 (3 marks)

Solve $4 - 8\sin^2 2\theta = 0$ for $0 \leq \theta \leq 2\pi$ **3**

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Question 23 (4 marks)

Consider the following bivariate data relating number of firefighters putting out a fire and on average how many square metres of damage was done.

No. of firefighters	4	10	16	30	100
Average of damage done (m^2)	98.5	271.1	550.6	1018.9	4414.8

a) Calculate the correlation coefficient *r* between the two variables.
Round your answer to the nearest 3 decimal places. **1**

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b) Consider the claim "having more firefighters causes larger scale fires".
Assess this claim with reference to the statement "correlation does not imply causation". **1**

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Q23 CONTINUED ON NEXT PAGE

c) Determine the equation of the least squares regression line, with coefficients correct to 2 decimal places.

Hence estimate to the nearest whole number how many firefighters are required to address 1500 m^2 of damage. **2**

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Question 24 (4 marks)

Events A and B are non-mutually exclusive, with $P(A) = 0.35$, $P(B) = 0.55$, and $P(\overline{A \cup B}) = 0.2$.

a) Draw a Venn diagram to represent the probabilities of events A and B, and find $P(A \cap B)$. The diagram should show the correct probability of each region. **2**

b) Determine $P(B|A)$, and determine if events A and B are independent. Explain your reasoning. **2**

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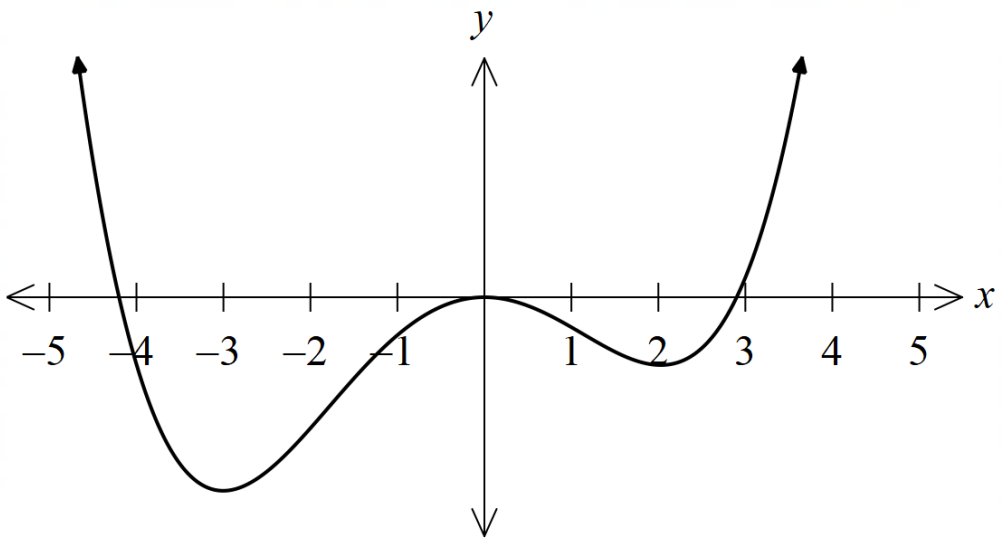
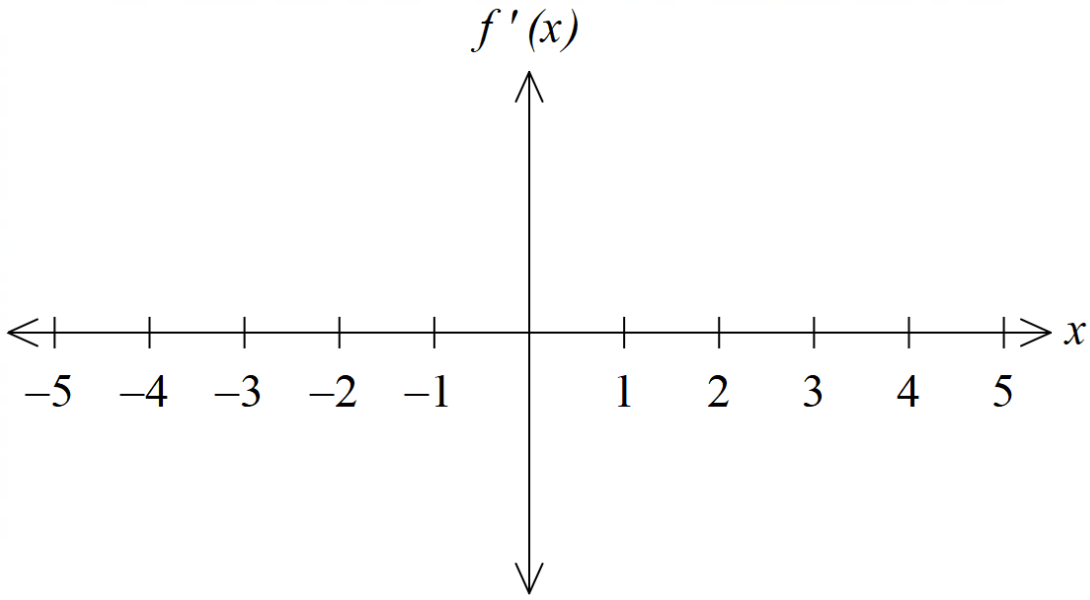
Question 25 (2 marks)

The diagram shows the graph of $y = f(x)$. **2**

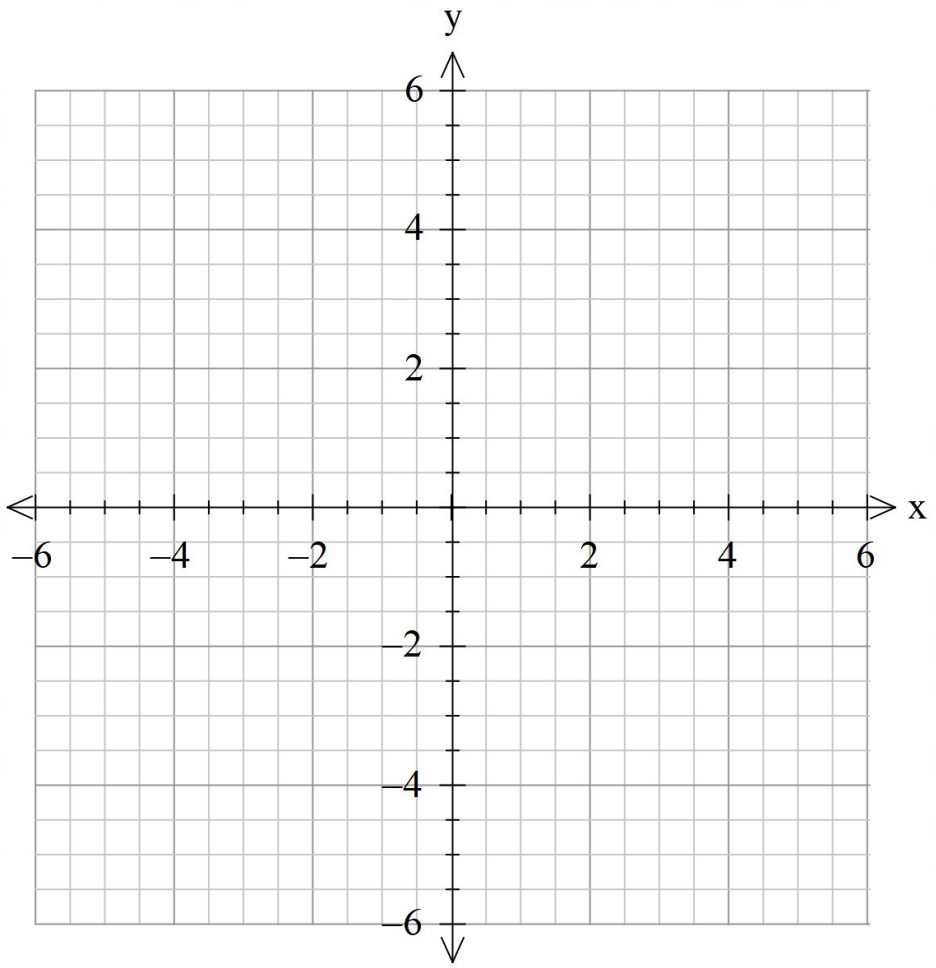


Sketch the graph of $y = f'(x)$ below.



Question 26 (5 marks)

a) Sketch the graph of $x^2 + y^2 - 4x = 0$, showing the coordinates of the centre and the radius. **3**



b) Hence, evaluate $\int_0^2 \sqrt{4x - x^2}\, dx$ **2**

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Question 27 (6 marks)

Let $f(x) = 10xe^{-x}$.

a) Find and determine the nature of any stationary points of $f(x)$. **3**

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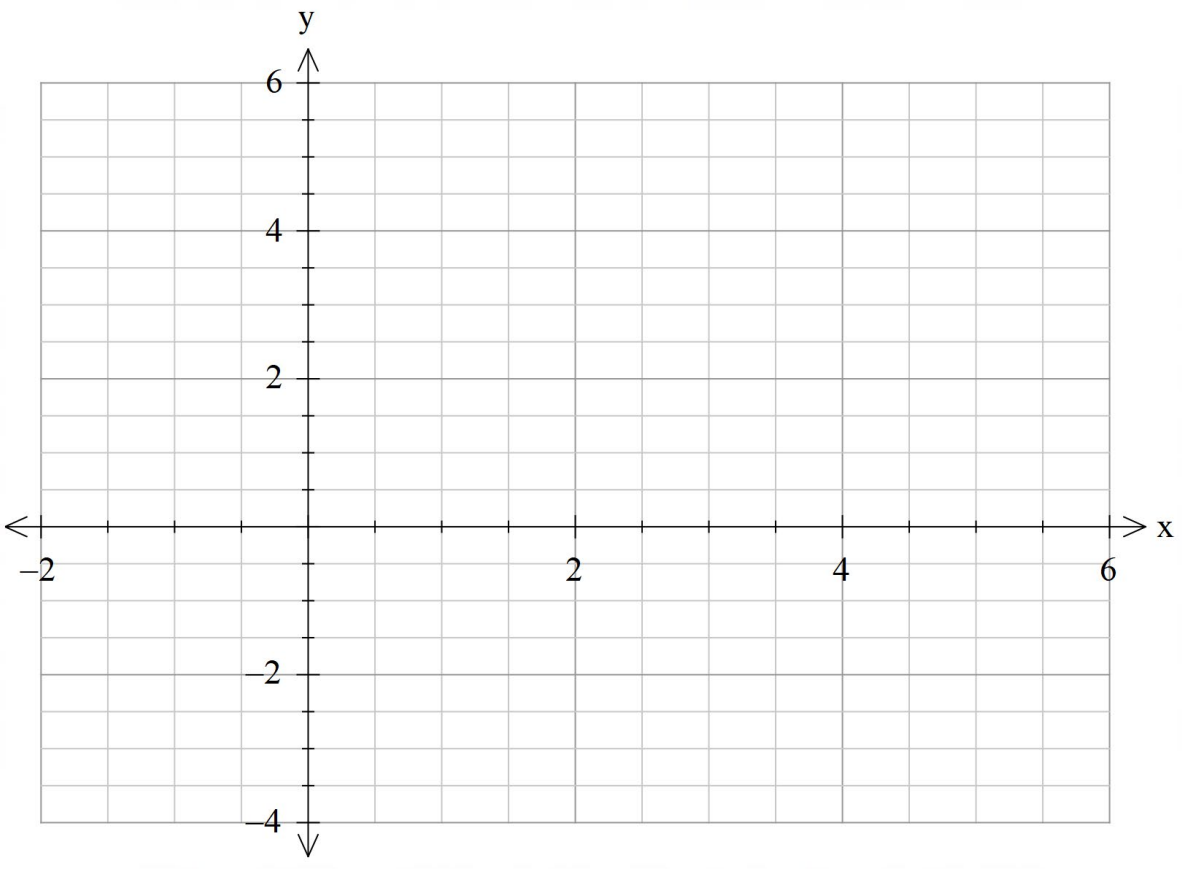
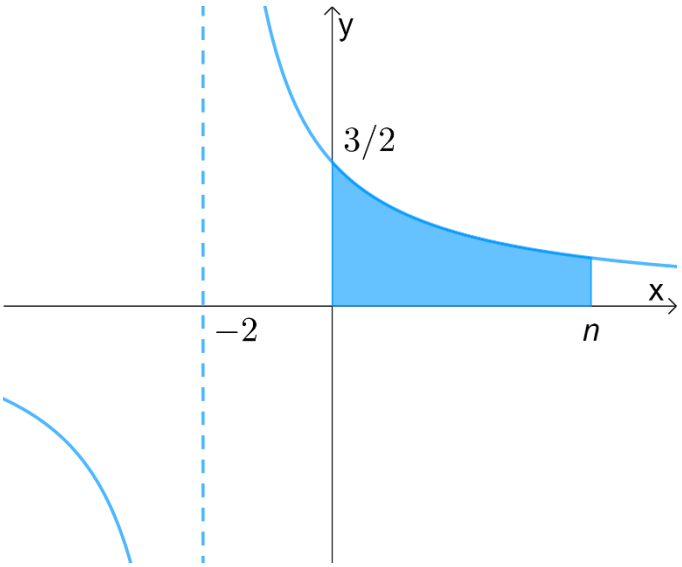
b) Given that there is a point of inflection when $x = 2$,
sketch the graph of $y = f(x)$, showing any stationary points, the point of inflection and any intercepts on the coordinate axes. **3**



Question 28 (4 marks)

Part of the graph of the function $f(x) = \dfrac{a}{x + b}$ is given **4**



If the shaded area is $\ln 27$, find the values of a, b *and* n.

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Write your student exam number in the boxes								

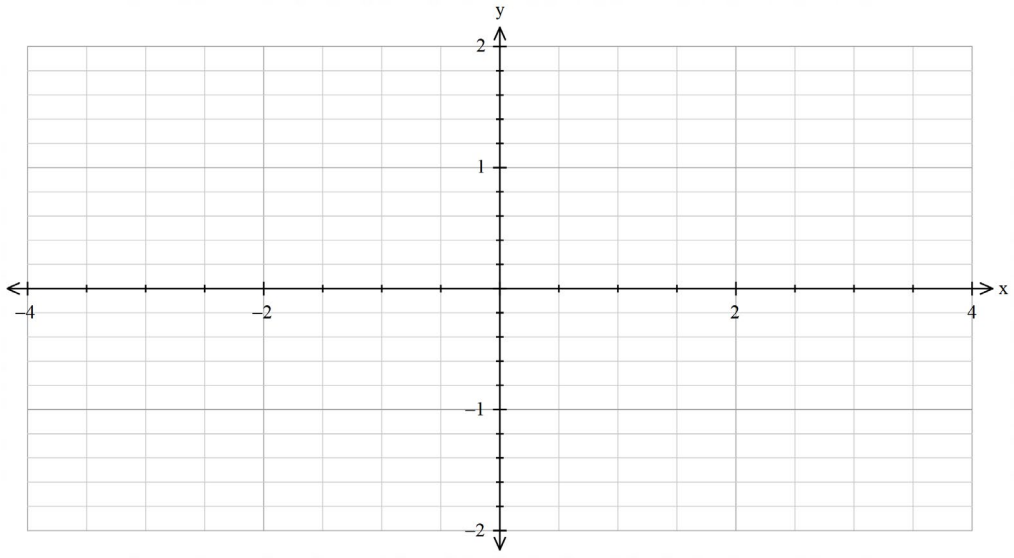
This Section II-E : Q 29 to Q 31

Indicate below if additional writing pages used.
Number of Additional Pages used.

Question 29 (3 marks)

a) On the axes below draw the graphs of

$$y = \sin\pi x \text{ AND } y = 1 - |x| \text{ for } -3 \leq x \leq 3$$ **2**



b) Hence find the number of solutions of the equation

$$\sin\pi x = 1 - |x| \text{ in the domain } (-\infty, \infty)$$ **1**

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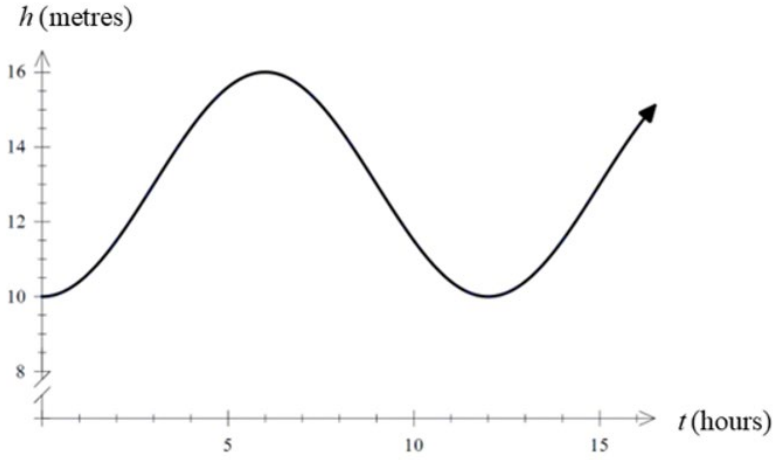
Question 30 (4 marks)

The rise and fall in sea level in Korsachov, due to tides, can be modelled by the cosine function below:

$$h(t) = A\cos(bt) + d$$



At 8am, on Tuesday, it is low tide, and the channel is 10m deep. At 2pm it is high tide, and the channel is 16m deep. A ship needs 11.5m of water to sail.

a) Find the values of A, b and d. **2**

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b) Between what times on Tuesday can the ship sail? **2**

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Question 31 (7 marks)

A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T°C, t minutes after it enters the liquid is given by,

$$T = 400e^{-0.05t} + 25, t \geq 0.$$

a) Find the initial temperature of the balls as it enters the liquid. **1**

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b) Find the value of t for which $T = 200$, giving your answer to 3 decimal places. **2**

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c) Find the rate at which the temperature of the ball is decreasing after 50 minutes, giving your answer to 3 decimal places. **2**

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Question 31 continues on page 25

Question 31 (continued)

d) As it cools the ball will eventually approach what temperature? **1**

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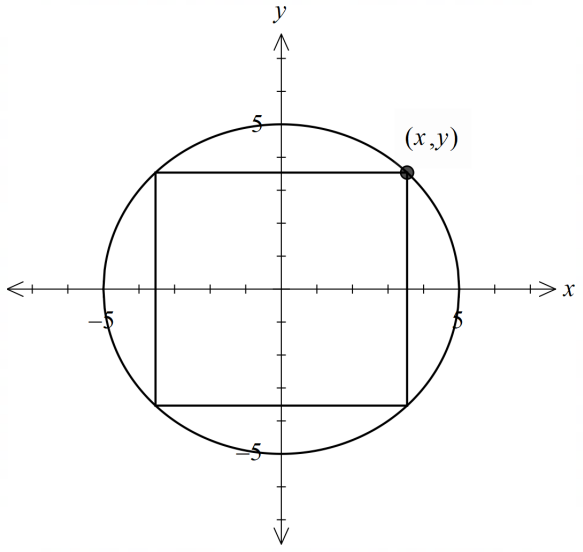
e) Sketch the graph of T. **1**

Question 32 (5 marks)

Consider a rectangle inscribed in a circle centred at the origin with a radius of 5, such that its upper-right corner is on the circumference of the circle at the point (x, y).



a) Show that the area of the rectangle is given by $A = 4x\sqrt{25 - x^2}$ **1**

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b) Find the dimensions of the rectangle such that it has the maximum possible area, and use calculus to show that this is the maximum possible area. **4**

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Question 33 (4 marks)

The probability density function of the normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The adult wingspan of an eagle species is normally distributed, with a mean of 180 cm and a standard deviation of 12 cm.

a) Use one application of the trapezoidal rule to find the approximate probability of randomly selecting one adult member of the species and finding its wingspan to be between 150 and 165 cm. Give your answer as a decimal correct to 3 decimal places. **3**

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b) Is the approximate probability found in part (a) an underestimate or an overestimate? Explain your reasoning. **1**

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Question 34 (8 marks)

A continuous random variable, X, has the following probability density function.

$$f(x) = \begin{cases} k \cos\left(\frac{\pi x}{2}\right) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $k = \frac{\pi}{2}$ **2**

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b) Find $P\left(X < \frac{1}{2} \mid X < \frac{2}{3}\right)$ **3**

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c) Find the median of X. **3**

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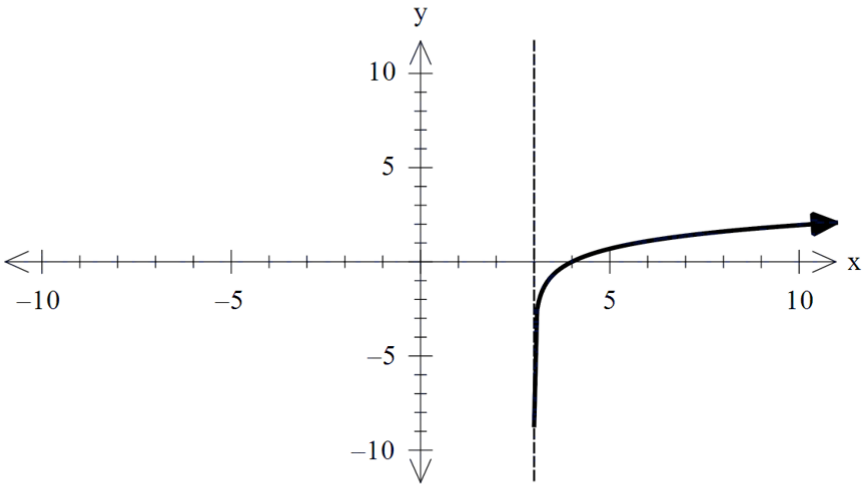
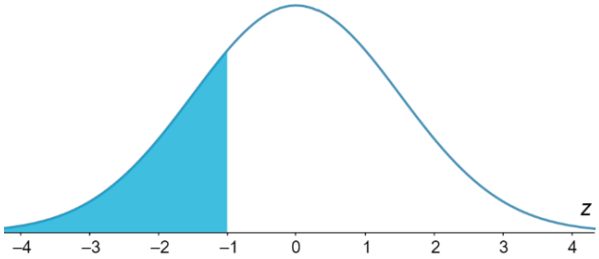
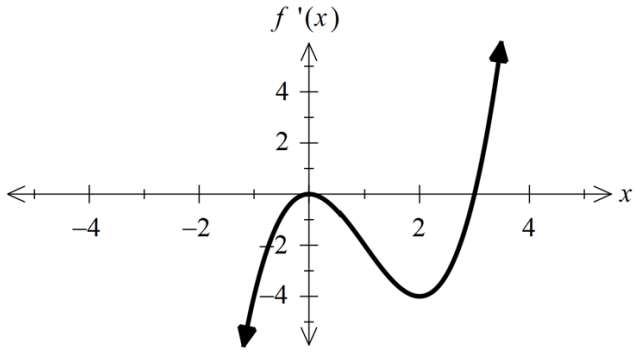
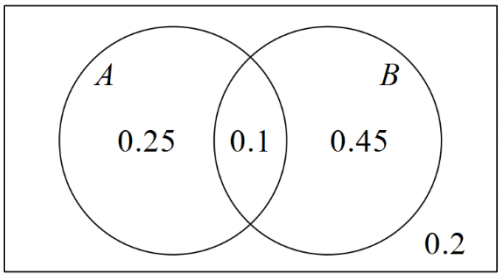
End of Paper

I 2021 HSC Trial exam -Mathematics Advanced (lockdown online examination)
Questions and solutions

Q		Ans
1	The graph of $f(x)$ is shown below, $x=3$ is an asymptote. What is the domain for $-f(-x)$?  A $D: x>3$ B $D: x<3$ C $D: x>-3$ D $D: x<-3$ (Reflect the curve in both axes.)	D
2	Which interval gives the range of the function $y=6\sin x-10$? A. $[6,-10]$ B. $[4,16]$ C. $[-16,-4]$ D. $[-4,16]$ Answer: $[-10-6,-10+6]=[-16,-4]$	C

3	The graph shows the area under a normal distribution curve.  Which of the following is equivalent to the shaded area? A. $P(z \geq -1)$ B. $P(z \geq 1)$ C. $1 - P(-1 \leq z \leq 1)$ D. $P(z \leq 1)$ Answer: $P(z \leq -1) = P(z \geq 1)$	B
4	Boxes are stacked in layers where each layer contains one box less than the layer below. There are six boxes in the top layer, seven boxes in the next layer, and so on. There are n layers altogether. Which of the following is the correct expression for the number of boxes in the bottom layer? A. $6n - 1$ B. $6n - 2$ C. $n + 5$ D. $n + 6$ **Answer:** $6 + 7 + 8 + \ldots$ $a = 6, d = 1$ $T_n = a + (n-1)d = 6 + 1(n-1) = 6 + n - 1 = n + 5$	C

5	Which of the following is the same as $\operatorname{cosec}(\pi + \theta)$? A. $-\frac{1}{\sin\theta}$ B. $-\frac{1}{\cos\theta}$ C. $\frac{1}{\cos\theta}$ D. $\frac{1}{\sin\theta}$ **Answer:** $cosec(\pi + \theta) = \frac{1}{\sin(\pi+\theta)} = \frac{1}{\sin\theta}$ (related angle identity)	A
6	What is the derivative of $y = 3^{2x+1}$? A. $y' = 2\ln 3 \cdot 3^{2x+1}$ B. $y' = \ln 3 \cdot 3^{2x+1}$ C. $y' = 2 \cdot 3^{2x+1}$ D. $y' = (2x+1) \cdot 3^{2x+1}$ (Use formula on reference sheet)	A
7	The graph of $y = f'(x)$ is shown below. For what values of x is the function $f(x)$ decreasing?  A. $(0,2)$ B. $(3,\infty)$ C. $(-\infty,3)$ D. $(-\infty,0)\cup(0,3)$ Answer: Look at where $f'(x) < 0$ (i.e. where the curve is below the x-axis)	D

8	What is the derivative of $\sin^2 3x$? A. $2\sin 3x$ B. $6\sin 3x$ C. $6\cos^2 3x$ D. $6\sin 3x\cos 3x$ Answer: $2\sin 3x \times \cos 3x \times 3$	D
9	Events A and B are non-mutually exclusive, with $P(A) = 0.35$, $P(B) = 0.55$, and $P(\overline{A \cup B}) = 0.2$. What is the value of $P(B\|A)$? A. $\frac{2}{7}$ B. $\frac{3}{7}$ C. $\frac{4}{7}$ D. $\frac{5}{7}$ Answer:  $P(A \cap B) = 0.35 + 0.45 + 0.2 - 1 = 0.1$ $P(B\|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.35} = \frac{2}{7}$	A
10	The graph of $y = \sqrt{x}$ undergoes the following sequence of transformations: - Translation 3 units right - Dilation horizontally by a factor of $\frac{1}{2}$ - Translation 1 unit down - Dilation vertically by a factor of 4 The equation of the resulting graph is: A. $y = 4\sqrt{2x - 3} - 1$ B. $y = 4(\sqrt{2x - 3} - 1)$	B

C. $y = 4\sqrt{\frac{x}{2} - 3} - 1$

D. $y = 4\left(\sqrt{\frac{x}{2} - 3} - 1\right)$

Answer:

$y = \sqrt{x-3}$

$y = \sqrt{2x-3}$

$y = \sqrt{2x-3} - 1$

$y = 4\sqrt{2x-3} - 4$ or $\sqrt{32x-48} - 4$

11

Marina scored the following results in her subjects.

Subject	Mark	Class Mean	Standard Deviation
Economics	64	60	3
Maths	71	60	10
Art	82	60	18
Chemistry	69	60	5

In which subject did she perform best in?

A Economics

B Maths

C Art

D Chemistry

D

Answer:

z-score for Economics: $\frac{64-60}{3} = 1.33\ldots$

z-score for Maths: $\frac{71-60}{10} = 1.1$

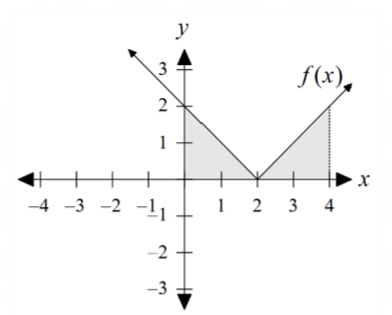
z-score for Art: $\frac{82-60}{18} = 0.66\ldots$

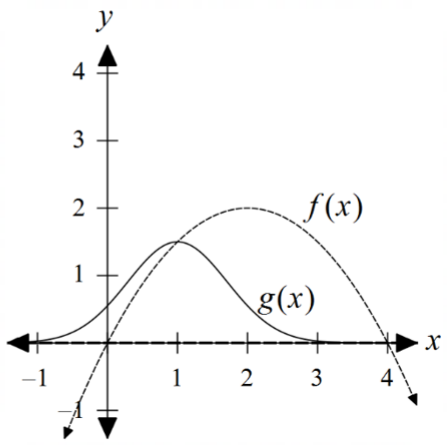
z-score for Chemistry: $\frac{69-60}{5} = 1.8$

12	How many solutions does $2\sin^2 x - 1 = 0$ have for $-\frac{\pi}{2} \le x \le \pi$? A. 1 B. 2 C. 3 D. 4 Answer: $\begin{cases} \sin x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4} \\ \sin x = -\frac{1}{\sqrt{2}} \Rightarrow x = -\frac{\pi}{4} \end{cases}$	C
13	The equation of the tangent to the curve $y = f(x)$ at the point (2,7) is $y = 5x - 3$. What is the equation of the tangent to the curve $y = f(x + 2) + 1$ at the point (0,8)? A. $y = 5x - 12$ B. $y = 3x - 4$ C. $y = 5x + 8$ D. $y = 5x - 13$ Answer: $y = 5x - 3$ translated 2 units left and 1 unit up will give $y = \left(5(x+2) - 3\right) + 1 \Rightarrow y = 5x + 8$	C

14	Which of the following is not a probability density function? A. $f(x)=\begin{cases}3x^2 & \text{for } 0\le x\le 1\\ 0 & \text{for all other values of } x\end{cases}$ B. $f(x)=\begin{cases}2x^4 & \text{for } 0\le x\le 1\\ 0 & \text{for all other values of } x\end{cases}$ C. $f(x)=\begin{cases}4x^3 & \text{for } 0\le x\le 1\\ 0 & \text{for all other values of } x\end{cases}$ D. $f(x)=\begin{cases}2x & \text{for } 0\le x\le 1\\ 0 & \text{for all other values of } x\end{cases}$ Answer: $\int_0^1 2x^4\,dx=\left[\frac{2x^5}{5}\right]_0^1=\frac{2}{5}\neq 1$	B
15	The value of $\int_1^3 \frac{x-1}{x^2-2x+5}\,dx$ is A) $\ln\sqrt{2}$ B) ln 2 C) ln 4 D) ln 8 Answer: $\frac{1}{2}\int_1^3 \frac{2x-2}{x^2-2x+5}\,dx$ $=\frac{1}{2}[\ln\lvert x^2-2x+5\rvert]_1^3$ $=\frac{1}{2}[\ln 8-\ln 4]$ $=\frac{1}{2}\ln 2$ $=\ln\sqrt{2}$	A

16	Which expression is a term of the geometric series $3x-6x^2+12x^3-\ldots\ldots$ A. $3072x^{10}$ B. $-3072x^{10}$ C. $3072x^{11}$ D. $-3072x^{11}$ Answer: $a = 3x, r = -2x$ $T_n = ar^{n-1} = 3x(-2x)^{n-1}$ Since $3072 = 3\times2^{10}$, $n = 11$ $T_{11} = 3x(-2x)^{10} = 3072x^{11}$	C
17	Find $\int \sin 2x\, e^{\cos 2x}\, dx$ A) $\frac{e^{\cos 2x}}{2} + C$ B) $-\frac{e^{\cos 2x}}{2} + C$ C) $e^{\cos 2x} + C$ D) $-e^{\cos 2x} + C$ Answer: $\int \sin 2x\, e^{\cos 2x} dx$ $= -\frac{1}{2}\int -2\sin 2x\, e^{\cos 2x} dx$ $= -\frac{1}{2}e^{\cos 2x} + C$, by the reverse chain rule	B

18	The graph of $f(x) = \|x - 2\|$ is shown.  Which of the following would give the area between $f(x)$ and the x-axis from $x = 0$ to $x = 4$, as shown by the shading on the graph? A. $\left\| \int_0^4 (x-2)dx \right\|$ B. $-\int_0^4 (x-2)\,dx$ C. $2\int_0^2 (x-2)\,dx$ D. $-2\int_0^2 (x-2)\,dx$ The graph is made up of two lines, the portion from $x = 0$ to $x = 2$ is $y = -x + 2$ and the portion from $x = 2$ to $x = 4$ is $y = x - 2$. Option A would produce an area of 0 by adding the negative triangle and positive triangle formed by $y = x - 2$. Option B would produce the same result as option A. Option C would give a correct area in absolute value by doubling the area of the triangle formed by $y = x - 2$ from $x = 0$ to $x = 2$, but it would produce a negative answer. Option D would give a correct area by doubling the area of the triangle formed by $y = x - 2$ from $x = 0$ to $x = 2$ and cancelling the negative. (This is the correct answer.)	D
19	Let $g(x) = 1 - f(x)$. If $\int_1^4 f(x)\,dx = 7$. What is the value of $\int_1^4 g(x)\,dx$? A. -6 B. -4 C. 8 D. 10	B

	Answer: $\int_1^4 g(x)\,dx$ $= \int_1^4 [1 - f(x)]\,dx$ $= \int_1^4 1\,dx - \int_1^4 f(x)\,dx$ $= [x]_1^4 - 7$ $= 4 - 1 - 7$ $= -4$	
20	The graphs of the functions $f(x)$ and $g(x)$ are shown.  A new function, $h(x)$, is defined $h(x) = g\big(f(x)\big)$. What is true about $h(3)$? A. $h(x)$ is decreasing at $x = 3$ B. $h(x)$ is increasing at $x = 3$ C. $h(x)$ has a local maximum at $x = 3$ D. $h(x)$ has a local minimum at $x = 3$ $h(3) = g(f(3))$. At this value of x, $f(3) \approx 1.5$, which makes $g\big(f(3)\big) = g(1.5) \approx 1$. As x increases, $f(x)$ decreases, which causes $g\big(f(x)\big)$ to increase. Further explanation: See the mentioned point on the graph at $g(1.5)$, and travel along the graph as x decreases (to the left) to see that g is increasing and is not near a stationary point.	B

Short Answer

Question 20 KL

Find the range of values of x for which the geometric series **2**

$$1 + \left(1 - \frac{x}{2}\right) + \left(1 - \frac{x}{2}\right)^2 + \left(1 - \frac{x}{2}\right)^3 \ldots\ldots$$

has a limiting sum.

$-1 < r < 1$ for series to have a limiting sum $r = 1 - \frac{x}{2}$ $-1 < 1 - \frac{x}{2} < 1$ $-2 < -\frac{x}{2} < 0$ $4 > x > 0$ $\therefore\ 0 < x < 4$	2 marks for correct solution fully demonstrated 1 mark - Correct expression for r - Correct calculation for x from incorrect expression for r - $0 \leq x \leq 4$

Question 23 CDS

Consider the following bivariate data relating number of firefighters putting out a fire and on average how many square metres of damage was done.

No. of firefighters	4	10	16	30	100
Average of damage done (m^2)	98.5	271.1	550.6	1018.9	4414.8

a) Calculate the correlation coefficient r between the two variables.
Round your answer to the nearest 3 decimal places. **1**

b) Determine the equation of the least squares regression line, with coefficients correct to 2 decimal places.
Hence estimate to the nearest whole number how many firefighters are required to address 1500 m^2 of damage. **2**

a) r = 0.998 (by calculator)	1 mark correct answer
b) From calculator i) D = 45.62 M –188.97 ii) $1500 = 45.62x - 188.97$ $x = 37.0225$ approx. 37 firefighters needed 38 was accepted	2 marks correct answer 1 mark correct substitution into incorrect equation 1 mark incorrect use of equation

Question 24 CD

A continuous random variable, X, has the following probability density function.

$$f(x) = \begin{cases} k\cos\left(\frac{\pi x}{2}\right) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Show that $k = \frac{\pi}{2}$ **2**

b) Find the median of X. **3**

$\int_0^1 k\cos\frac{\pi x}{2}\,dx = 1$ $k\left[\frac{2}{\pi}\sin\frac{\pi x}{2}\right]_0^1 = 1$ $\frac{2k}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right) = 1$ $k = \frac{\pi}{2}$	2 marks for correct solution 1 mark for stating correct integral equality involving k (i.e. Line 1)
$\int_0^m \frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)dx = 0.5$ $\left[\sin\left(\frac{\pi x}{2}\right)\right]_0^m = 0.5$ $\sin\left(\frac{\pi m}{2}\right) = 0.5$ $\frac{\pi m}{2} = \frac{\pi}{6} \Rightarrow m = \frac{1}{3}$	3 marks for correct solution 2 marks - finding the correct trigonometric equation (i.e. Line 3) - solving an incorrect trig integral equated to 0.5, e.g. $\left[-\sin x\left(\frac{\pi x}{2}\right)\right]_0^{\pi} = 0.5$

	1 mark for stating correct integral equation involving median variable (i.e. Line 1)